

UNIVERSITY OF CRAIOVA
Faculty of Mathematics and Computer Science
Department of mathematics
Fundamental domain : Exact sciences
Domain: Mathematics
Master: Applied mathematics
Form: cursuri de zi
Duration of studies: 2 years
Approved with academic year 2008-2009

Ordered algebraic structures

Syllabus

Course coordinator: Prof. dr. Dumitru Buşneag
Cod :MA124
Second Cycle: MASTER
First Year , Semester 2, Course 28 hours, Seminar 28 hours
Nr. of credits: 6
Domain: Mathematics
Type : compulsory
Category: complementary

Objectives : To enable the students with fundamentals results from theory of ordered groups and rings, the study of some categories of lattices and classic categories of algebras of logic (Stone,Boole, Heyting,Hilbert,etc).

Necessary background : Courses of Algebra, Logic and theory of sets, Mathematical analysis and Topology from Licence cycle and Topics in theory of categories , First Year, semester 1 from this master.

Evaluation : Exam (E).

Contents:

A. Ordered sets. Semilattices. Lattices. Ideals and filters in a lattice.Modular lattices. Distributive lattices. Prime (maximal) ideals(filters) in a distributive lattice.Theorem of prime ideal(filter) in a bounded distributive lattice. Complemented and pseudo-complemented lattices. Boolean algebras. Heyting algebras. Categorical study of these structures.

B. Ordered groups. Convexe subgroups. Latticial ordered groups.Morphisms of latticial ordered groups. Absolute values and orthogonality. Polars.Reprezentative groups.

C. Archimedean ordered structures. Total ordered rings and fields. Archimedean ordered fields.Archimedean total ordered group.

D. Ordered rings. Ordered and latticial rings. Examples. l-ideals in a latticial ring. .Morphisms. Quotient latticial ordered ring . Subdirect products of latticial rings.Irreducible, prime and semi-prime L-ideals.Directe subproducts of total ordered rings.

E. Another ordered algebras of fuzzy logic: MV-algebras, BL-algebras,Hilbert and Hertz algebras.

Bibliography

1. R.Balbes, Ph. Dwinger : *Distributive lattices*, University of Missouri Press, 1974.
2. T.S. Blyth: *Lattices and ordered algebraic structures*, Springer, 2005.
3. A. Bigard, K. Keimel, S. Wolfenstein: *Groupes et anneaux ordonne*, Lectures Notes in Mathematics, 608, Springer, Berlin, 1971.
4. D. Busneag: *Categories of algebraic logic*, Ed. Academiei, Bucuresti, 2006.
5. G.Gratzer: *Lattices theory*, W.H.Freeman and Company, 1971.
6. S. M. Lane: *Categories for the Working Mathematician*, Springer,1997.
7. Gh. Radu: *Algebra categoriilor si a functorilor*, Ed. Junimea, Iasi, 1988.
8. E. Turunen: *Mathematics behind Fuzzy logic*, Physica-Verlag, 1999.